Vibration damping of structures coupled to passive piezoelectric networks

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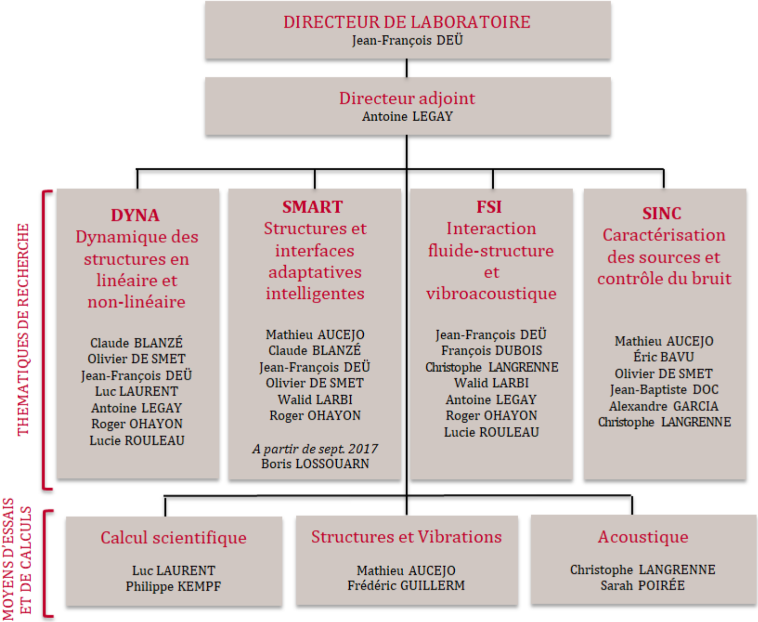
and co-authors from

Conservatoire national des arts et métiers
École normale supérieure de Rennes
Georgia Institute of Technology
University of Liège
École Navale...

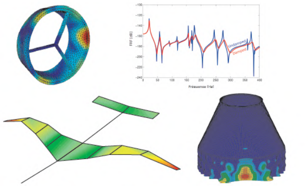
Séminaire du département Mécatronique, ENS Rennes, 11 février 2020

le cnam

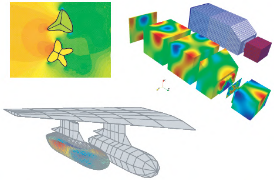
Laboratoire de Mécanique des Structures et Syst. Couplés



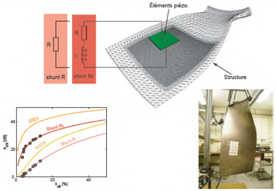
Thématiques de recherche du LMSSC



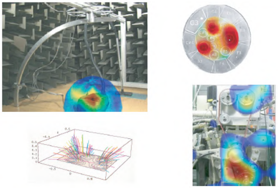
Dynamique des structures en linéaire et non-linéaire



Interaction fluide-structure et vibro-acoustique



Structures et interfaces adaptatives intelligentes



Caractérisation des sources et contrôle du bruit

Projets et collaborations

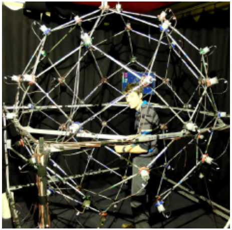
- Projets industriels
  - ARIANE GROUP, SAFRAN, AIRBUS, NAVAL GROUP...

- Collaborations recherche avec
  - des laboratoires de recherche en France
  - des laboratoires de recherche à l'étranger
  - des organismes de recherche : DGA, CNES, CSTB, ...

- Projets transverses avec des partenaires d'autres disciplines
  - Modélisation mécanique pour l'archéologie virtuelle : restitution d'un char celtique à deux roues - Coll. avec le Musée d'Archéologie de Saint-Germain

Organisation d'évènements pour le grand public

- Quelques exemples:
  - Nuit européenne des musées : Immersion sonore 3D
  - Fêtes de la science avec le Musée des arts et métiers : Conception d'un aéronef; Bruit et vibrations : du réel au virtuel; ...

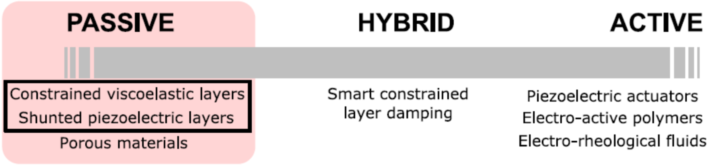


Outline

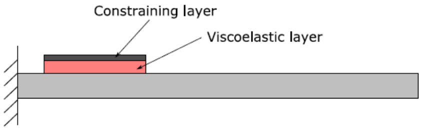
1. Laboratoire de Mécanique des Structures et des Systèmes Couplés
2. Piezoelectric tuned vibration absorber
3. Finite element models and optimization for complex structures
4. Environmental parameters: beyond the linear shunt
5. Multimodal vibration damping
6. Conclusions and perspectives

Outline

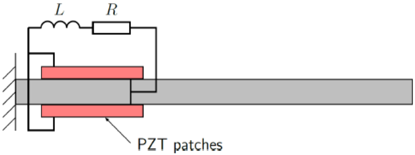
Passive vibration mitigation



- Constrained viscoelastic patches



- Piezoelectric patches connected to an electrical network

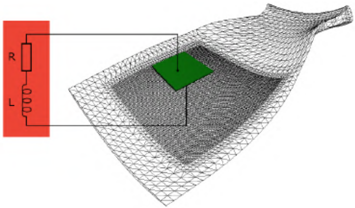


(✍ Lucie Rouleau)

Piezoelectric damping offers great perspectives

Vibration reduction for structural integrity and increased lifespan

→ Ex.: Turbofan engine (✍ Sénéchal, 2011 + Thierry, 2016 / Safran)



Piezoelectric patch connected to a **passive electrical circuit**

→ **Resonant shunt** = Inductor + Resistor (✍ Hagood & von Flotow, 1991)

→ Electrical resonance tuned to a **single and linear mechanical mode**

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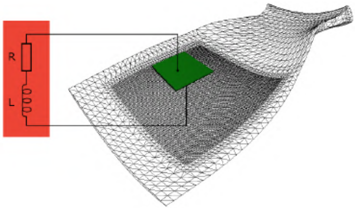

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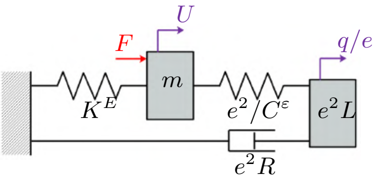
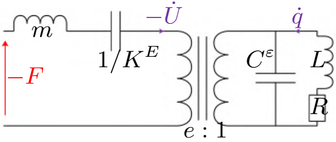
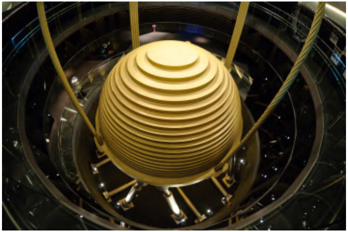
Electromechanical Tuned Mass Damper







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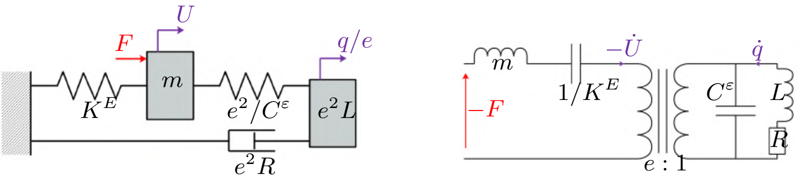


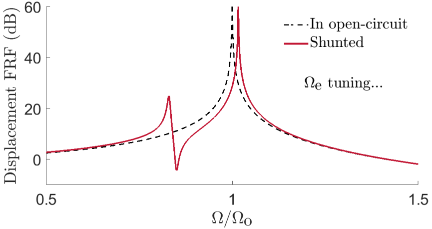
$$\Longrightarrow \quad \left(\frac{U}{F}\right)_{\text{adim.}} = \frac{1+2\mathrm{j}\frac{\xi_e}{\Omega_e}\Omega-\left(\frac{\Omega}{\Omega_e}\right)^2}{\left[1-\left(\frac{\Omega}{\Omega_o}\right)^2\right]\left[1+2\mathrm{j}\frac{\xi_e}{\Omega_e}\Omega-\left(\frac{\Omega}{\Omega_e}\right)^2\right]-\frac{k_{c0}^2}{1+k_{c0}^2}}$$

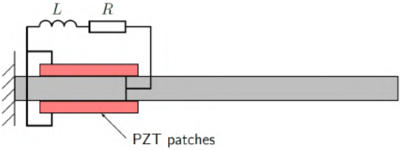
Coupling coefficient k_{c0} Mechanical resonance Ω_o

Electrical damping ξ_e Electrical resonance $\Omega_e = \frac{1}{\sqrt{LC^\varepsilon}}$

Tuning of the resonant shunt

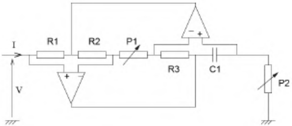
Transfer function criterion: $(\Omega_e)_{opt} = \Omega_o$ $(\xi_e)_{opt} = \sqrt{\frac{3}{8}} k_c$

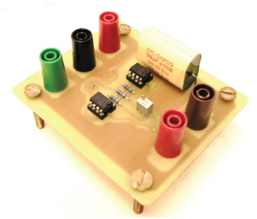




Several solutions to obtain large inductance values

Synthetic inductor
with analog circuit
(✍ Antoniou, 1969)





$$\mathrm{j}\omega C_1 P_2 R_3 \frac{R_1}{R_2} - P_1 \frac{R_1}{R_2}$$

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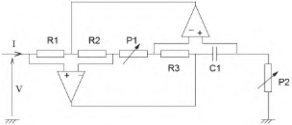
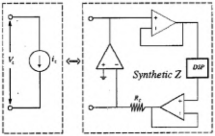
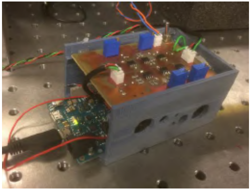


$$j\omega C_1 P_2 R_3 \frac{R_1}{R_2} - P_1 \frac{R_1}{R_2}$$

Synthetic impedance
with digital controler
(✍ Fleming, 2002)





$$Z(\omega)$$

Several solutions to obtain large inductance values

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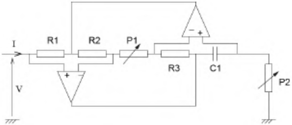

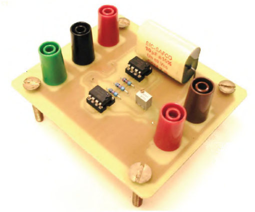

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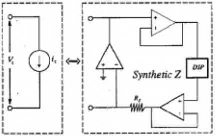
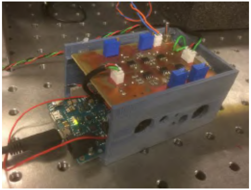
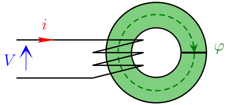




$$Z(\omega)$$

Passive inductor
with magnetic core

Sensors & Actuators, 2017

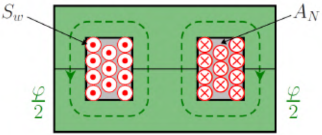


up to 1000 H !

$$\mathrm{j}\omega L_{eq}(\omega) + R_{eq}(\omega)$$

Resonant shunt → **Specifications on L and R**

Restriction on the available room



Window utilization factor

$$k_u = \frac{N S_w}{A_N} \approx 0.5 \text{ when full}$$

Magnetic cores with high permeance ($A_L \geq 10$ μH)

→ **Cores in ferrite** or **Nanocrystalline toroids** (Vitroperm 500F)

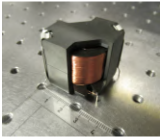
\+ **Energy** considerations...

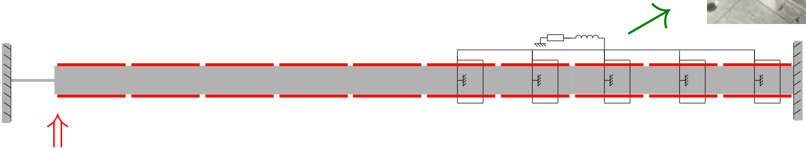
Resonant shunts can be implemented with passive inductors

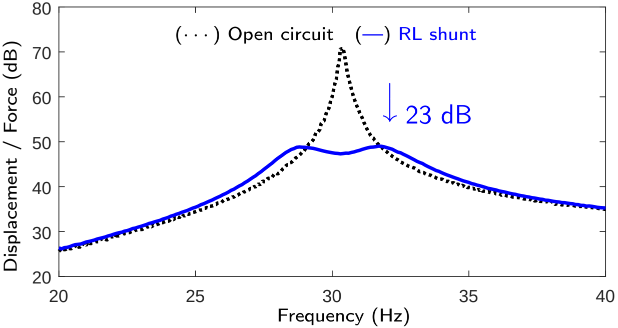
Realization with a **magnetic core** in ferrite (100 H)

→ ✍ *B. Lossouarn, M. Aucejo, J.-F. Deü, B. Multon, Sensors & Actuators A, 2017*

→ $C = 250$ nF ⇒ $L = 100$ H & $R = 3\text{k}\Omega$





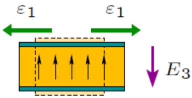


Outline

From linear piezoelectricity to a finite element model

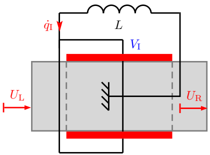
▷ **Constitutive law :**

$$\begin{cases} \sigma_i = c^E_{ij}\varepsilon_j - e_{ji}E_j \\ D_i = e_{ij}\varepsilon_j + \epsilon^{\varepsilon}_{ij}E_j \end{cases}$$



"31" effect

- c^E_{ij} : stiffness const. (with constant E)
- e_{ij} : piezoelectric const.
- $\epsilon^{\varepsilon}_{ij}$: permittivity (with constant ε)



Mass, Stiffness and Coupling matrices

$$\begin{pmatrix} \mathbf{M}_m & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{V}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_m & \mathbf{K}_c \\ -\mathbf{K}_c{}^{\mathsf{T}} & \mathbf{K}_e \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{Q} \end{pmatrix}$$

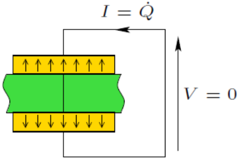
\+ **Shunt impedance** equation: $V = \omega^2 LQ$

Coupling coefficient from open- and short-circuit conditions

▷ $\mathbf{V} = 0$, **short-circuit**

$$\mathbf{M}_m \ddot{\mathbf{U}} + \mathbf{K}_m \mathbf{U} = \mathbf{0}$$

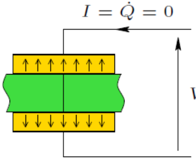
⇝ $(\omega_{\mathrm{sc},i}, \boldsymbol{\Phi}_{\mathrm{sc},i})$, short-circuit (sc) eigenmodes



▷ $\mathbf{Q} = 0$, **open-circuit**

$$\mathbf{M}_m \ddot{\mathbf{U}} + \left(\mathbf{K}_m + \mathbf{K}_c \mathbf{K}_e^{-1} \mathbf{K}_c^{\mathsf{T}}\right) \mathbf{U} = \mathbf{0}$$

⇝ $(\hat{\omega}_{\mathrm{oc},i}, \hat{\boldsymbol{\Phi}}_{\mathrm{oc},i})$, open-circuit (oc) eigenmodes



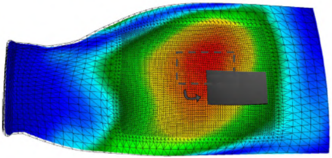
Performance and electrical tuning directly related to the **coupling coefficient**

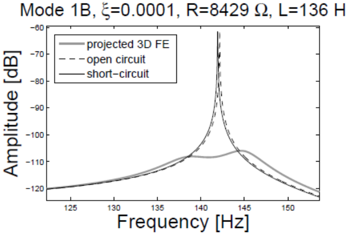
$$k_c = \sqrt{\frac{\omega_{OC}^2 - \omega_{SC}^2}{\omega_{SC}^2}} \quad \Rightarrow \quad \boldsymbol{L = \frac{1}{C\omega_O^2}} \quad \text{and} \quad \boldsymbol{R = \sqrt{\frac{3}{2}}\frac{k_c}{C\omega_O}}$$

Optimization for maximizing the coupling coefficient

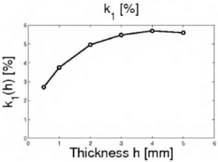
▷ **Optimization results :** 1B

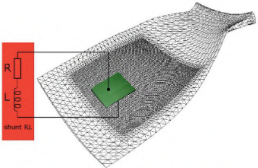
$P = 1$, fixed width and length

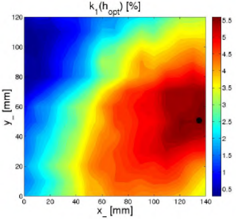


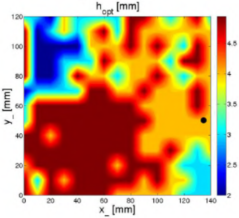


▷ **Exhaustive search :** 1B





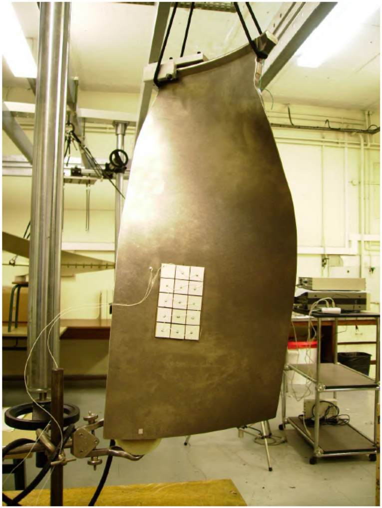


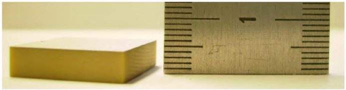


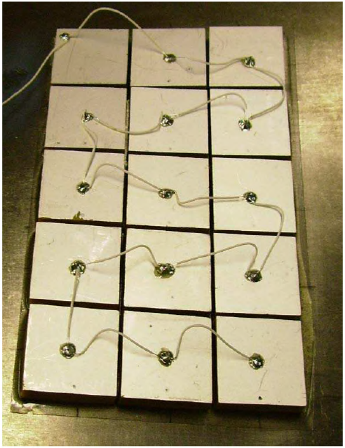
(✍ Sénéchal, 2011)

Current developments : uncoupled optimization for thin piezoelectric patches

First experiments on titanium blades (CFM56)





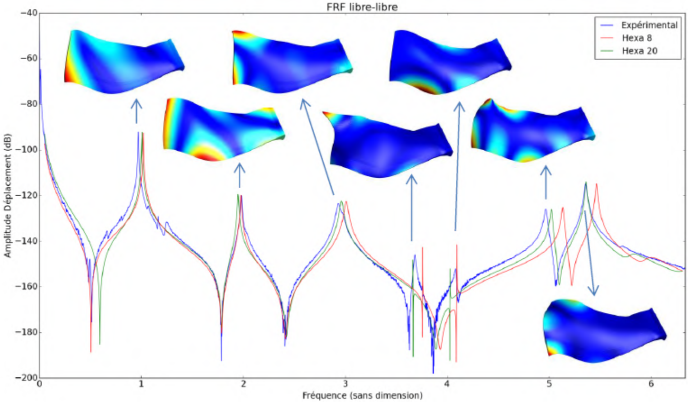


Proof of concept for piezoelectric damping of an industrial structure (-24 dB)

Extension to woven carbon-epoxy fan blades (LEAP)

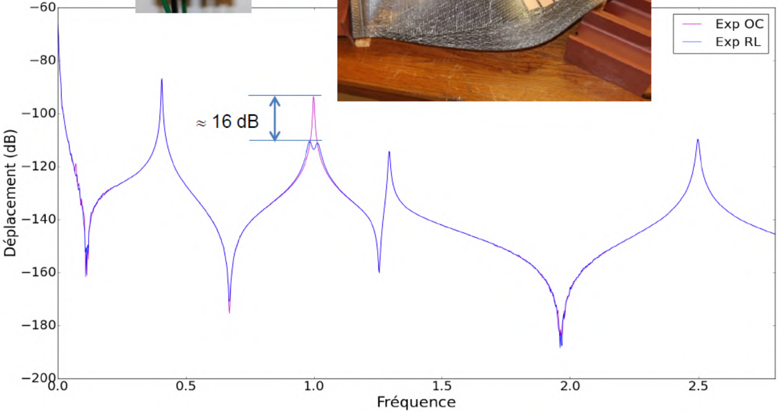
Mode shapes identification to define positioning of thin PZT patches

(✍ Thierry, 2016)



Experimental validation of vibration mitigation performance

With purely passive inductance



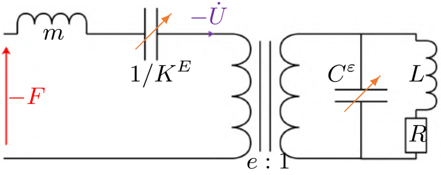
Significant damping with less than 1% mass addition (PZT = 0.2 mm)

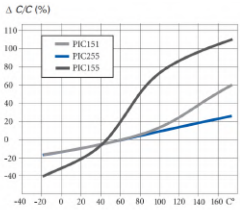
Outline

Temperature has a non-negligible influence

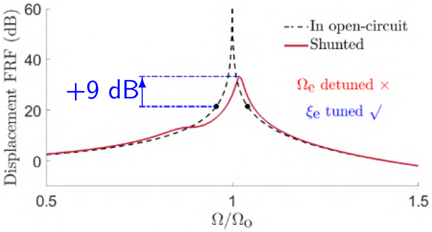
In case of temperature variations:

$\Omega_e(T) = \Omega_o(T)$





Datasheets: P.I. Ceramic

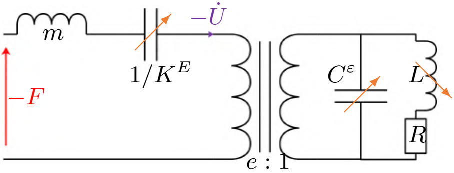


$\Omega_e = 0.9\ \Omega_o$

$\xi_e = (\xi_e)_{opt}$

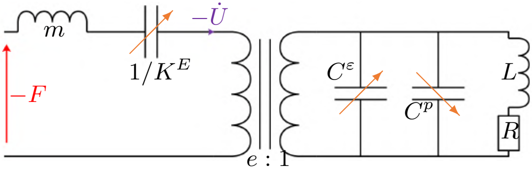
$k_{c0} = 0.12$

Using a variable inductance or a variable capacitance ?



Tuning condition: $\Omega_o^2(T) = \frac{1}{L(T)C^\varepsilon(T)}$

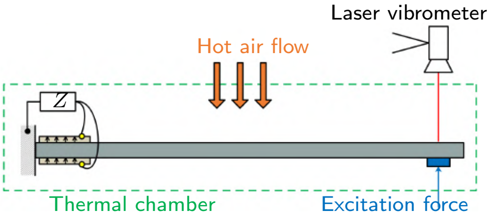
More details: *R. Darleux et al., JSV, 2018*

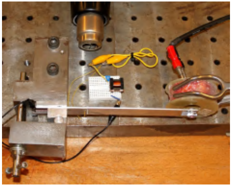


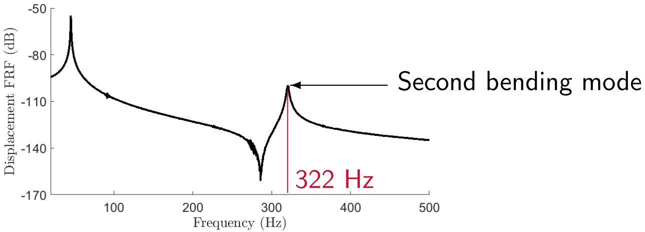
Tuning condition: $\Omega_o^2(T) = \frac{1}{L[C^\varepsilon(T)+C^p(T)]}$

Experiments to extract temperature dependence

Temperature range: Room temperature: 22 °C $\Longrightarrow$ 60 °C

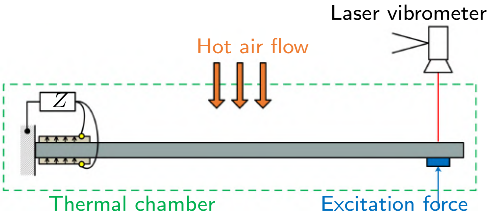


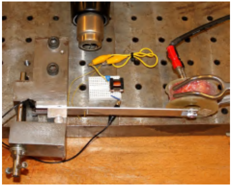


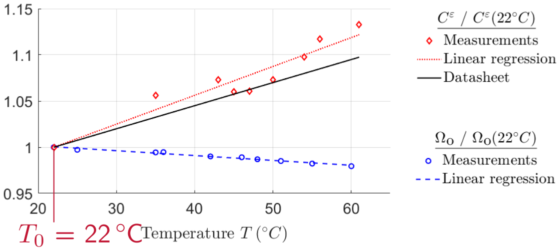


Experiments to extract temperature dependence

Temperature range: Room temperature: 22 °C $\Longrightarrow$ 60 °C



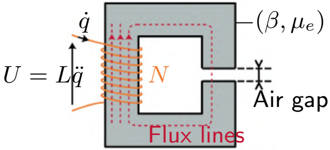




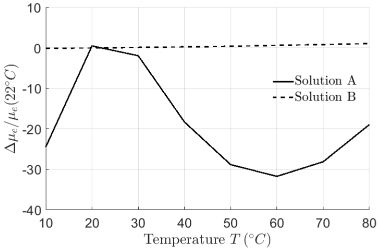
Design of passive inductors

$$L = \beta \mu_e N^2$$

β : geometric constant
μ_e : effective magnetic permeability
N : turns

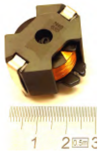


Datasheet: Epcos TDK



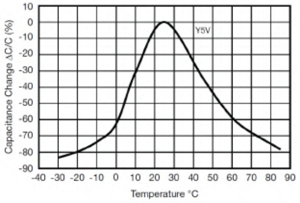
Solution A

Solution B

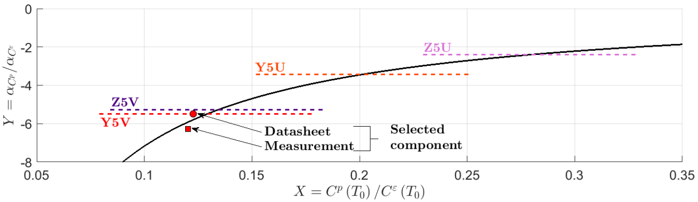


Choice of a variable capacitor

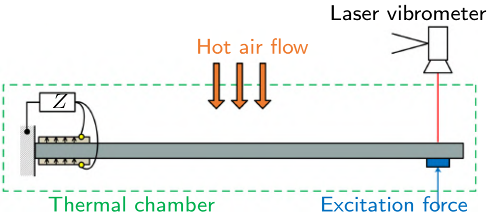
Datasheets: Vishay BCcomponents



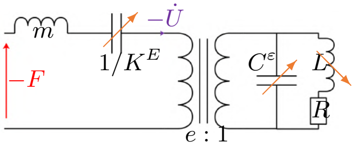
Class 2 ceramic capacitors
Existing dielectrics: Y5U, Y5V, Z5U, Z5V...



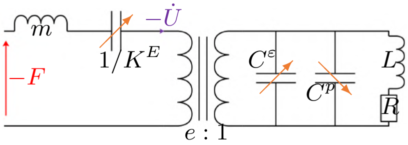
Vibration damping of a clamped beam



Solution A

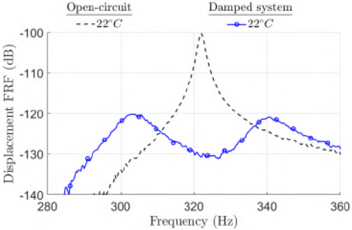


Solution B

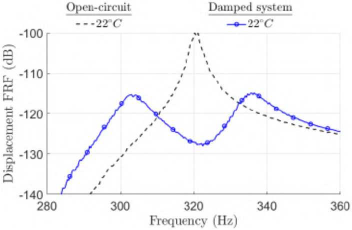


Comparison of adaptive resonant shunt solutions

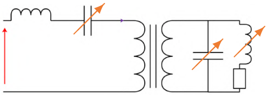
Solution A

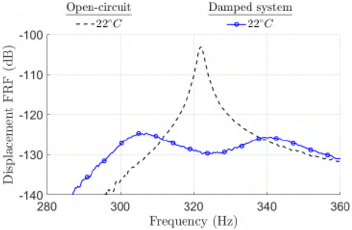


Solution B



Without adaptive tuning





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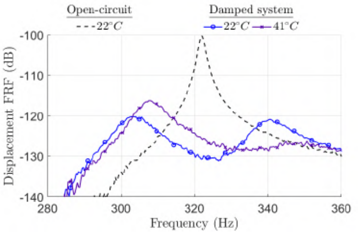
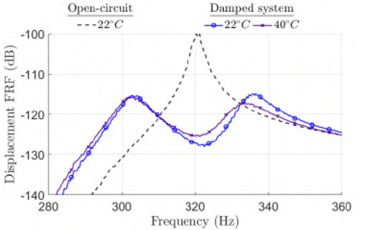
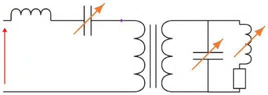
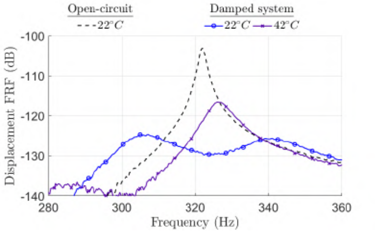


Solution B



Without adaptive tuning





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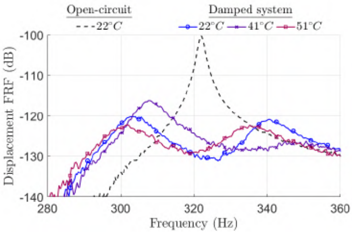
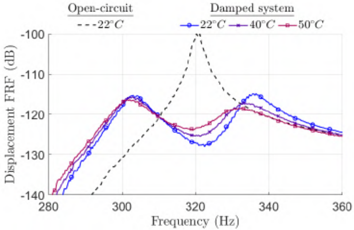
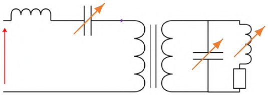
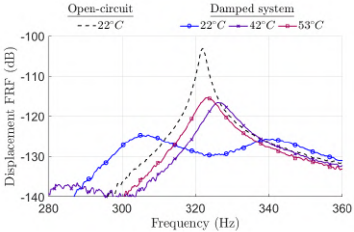


Solution B



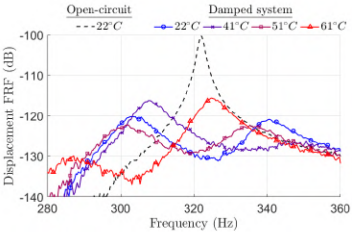
Without adaptive tuning



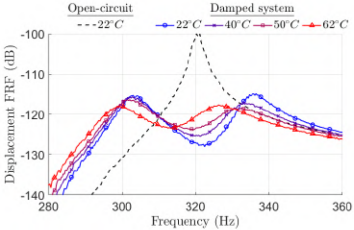


Comparison of adaptive resonant shunt solutions

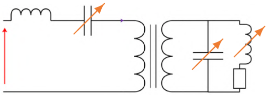
Solution A

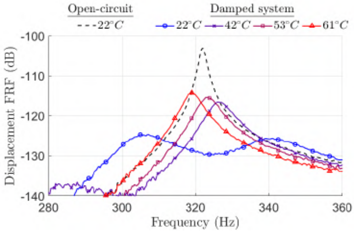


Solution B



Without adaptive tuning





Comparison of adaptive resonant shunt solutions

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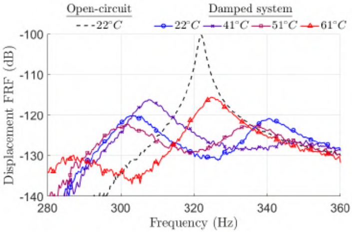
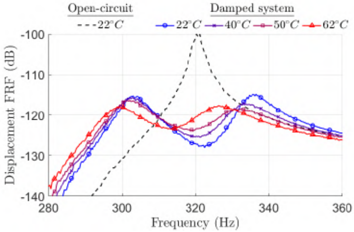
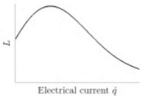


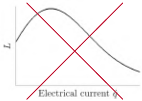
Solution B



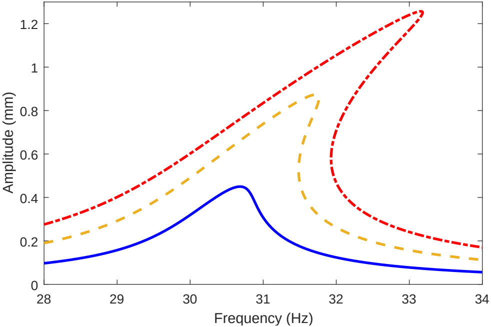
Damping performance maintained on a given temperature range

Excitation dependent tuning | Tuning ∼ independent from the excitation





Most industrial structures are nonlinear



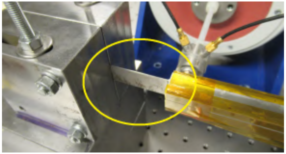
Objective of this study:
Linear + Nonlinear
"Mirror"

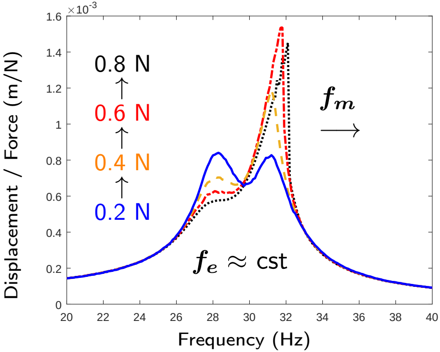
Multimodal & Nonlinear
Mechanical structure

Multi-resonant & Nonlinear
Electrical network

Yet, nonlinearity strongly affects the performance

Thin lamina $\Rightarrow f \approx K_{\mathrm{L}}u + K_{\mathrm{NL}}u^3$
$\rightarrow$ **Hardening nonlinearity**
$\rightarrow$ **Detuning** of the resonant shunt





$\rightarrow$ **Nonlinear piezoelectric tuned vibration absorber required !**

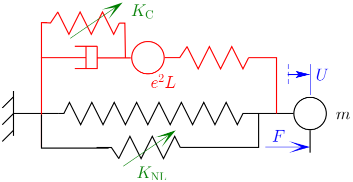
Similar nonlinearity in the absorber for global compensation

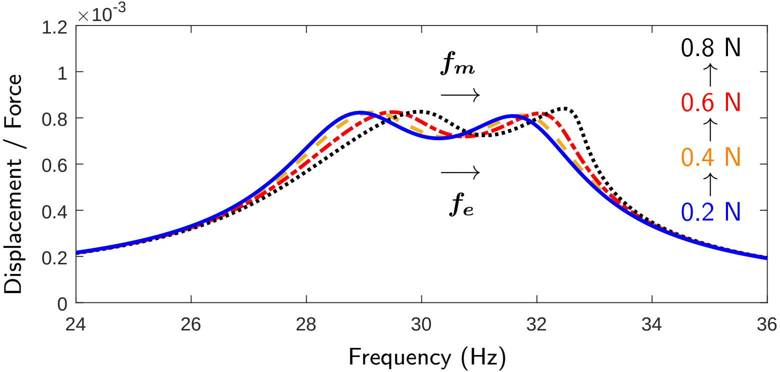
"Nonlinear + Nonlinear = Linear" (✍ Habib, 2016)

Tuning rule (✍ Soltani, 2015)

$$\rightarrow K_C \simeq 2\left(\frac{e^2 L}{m}\right)^2 K_{\mathrm{NL}}$$

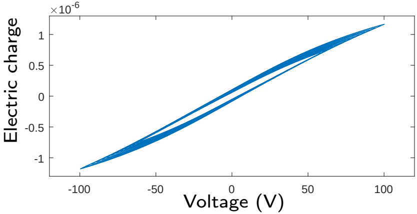
for cubic nonlinearity



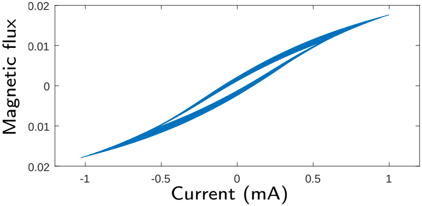


How to implement the nonlinearity in the electrical domain ?

Capacitor: $\boldsymbol{Q = CV}$



Inductor: $\boldsymbol{\Phi = L\dot{Q}}$

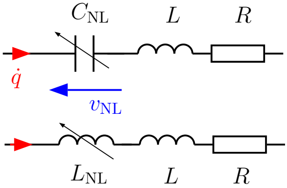


Nonlinear capacitance

$$C_{\mathrm{NL}} = C^{\star}/q^2 \quad \Rightarrow \quad v_{\mathrm{NL}} \;\; \alpha \;\; q^3$$

Nonlinear inductance

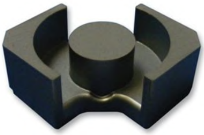
$$L_{\mathrm{NL}} = -L^{\star}\dot{q}^2 \quad \Rightarrow \quad v_{\mathrm{NL}} \;\; \alpha \;\; -\dot{q}^2\ddot{q}$$

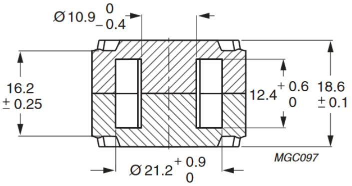


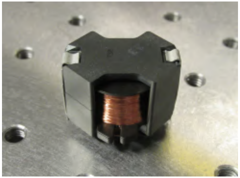
$\rightarrow$ Same cubic voltage after one-term Harmonic Balance approximation

Inductor design from magnetic component theory

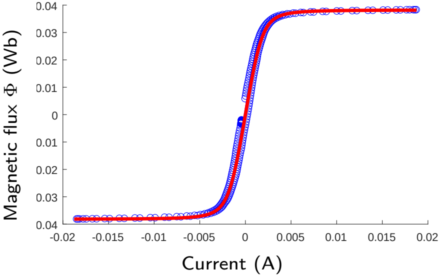
Magnetic saturation: Depends on material, geometry, number of turns...

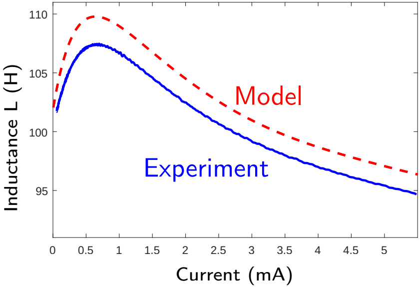




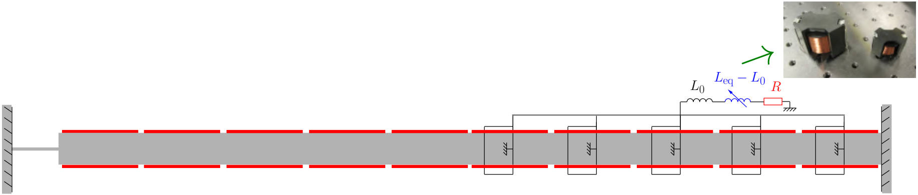


Relation between total magnetic flux and electrical current $\Rightarrow$ $\boldsymbol{\Phi = L\dot{Q}}$



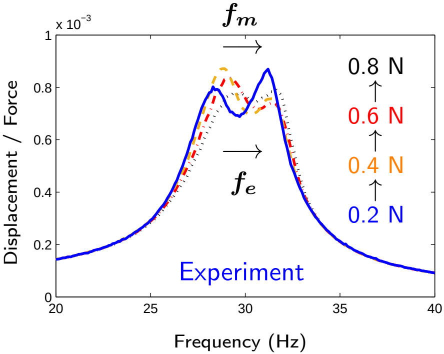


Intentional nonlinearity in the piezoelectric absorber



A fully passive solution:
Nonlinear inductor

→ Variation of the inductance value due to **magnetic saturation**



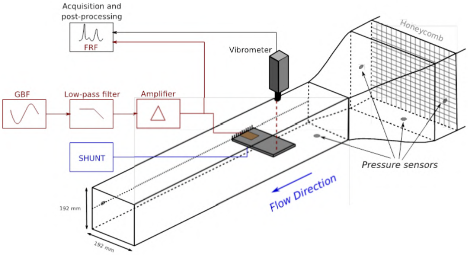
→ ✍ *B. Lossouarn, J.-F. Deü, G. Kerschen, Philosophical Transactions of the Royal Society A, 2018*

Flow-induced vibrations can have dramatic consequences

→ Structural and acoustic issues

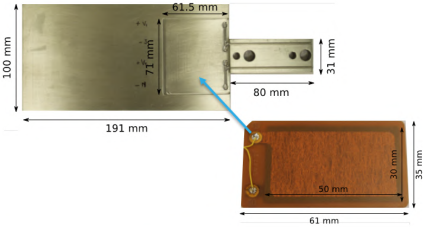
Hydrodynamic test facility at IRENav

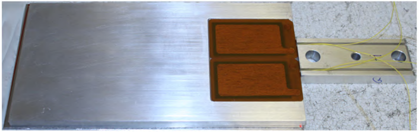
Experimental setup:



- Cantilevered aluminium flat plate
- Incidence of 0°
- 2 patches on each side

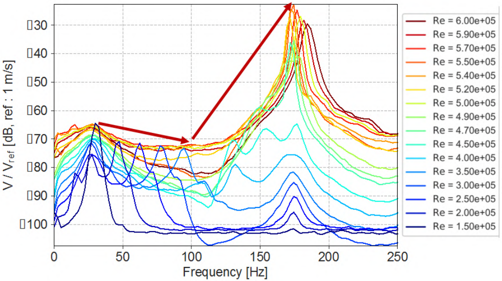
Geometry:





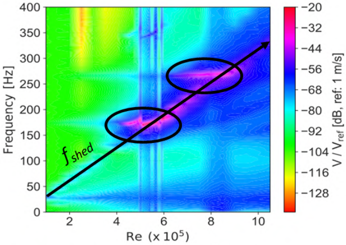
(✍ Laetitia Pernod)

Frequency response for different flow velocities:

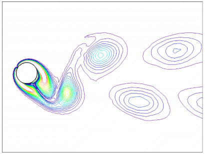


- Natural frequencies observed
- Additional components with a strong dependance to the velocity

Von Kármán vortex shedding:



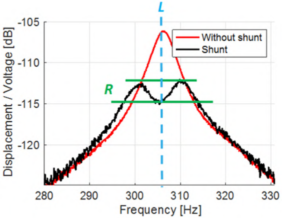
- Strouhal's law: $S_t = \frac{f_{shed} . e}{U_0} \sim 0.2$

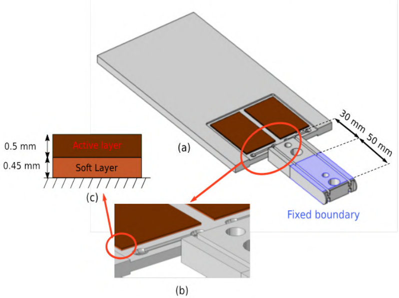


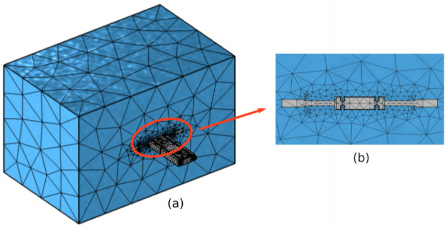
Numerical model involving surrounding fluid

Numerical simulation with COMSOL Mutliphysics

Structural model + Piezoelectricity + Acoustics → Natural frequencies + Coupling factors →

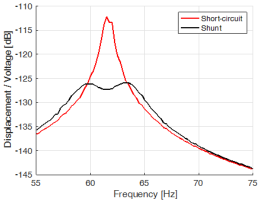




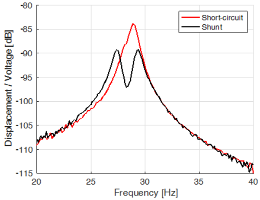


Performance of the piezoelectric shunt

Damping using resonant shunt (1st bending mode)

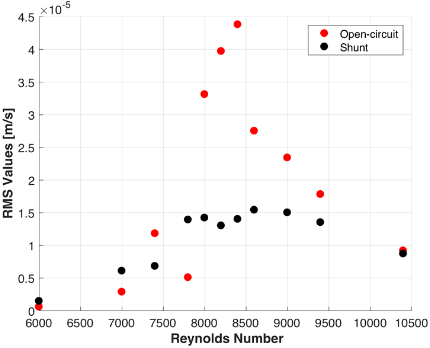


In air



In still water

Vibration reduction under hydrodynamic flows

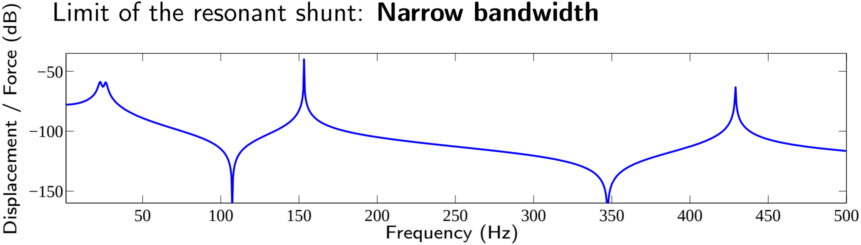


RMS value divided by 3

Outline

Passive technique for multimodal damping ?

Limit of the resonant shunt: **Narrow bandwidth**



Interconnected array

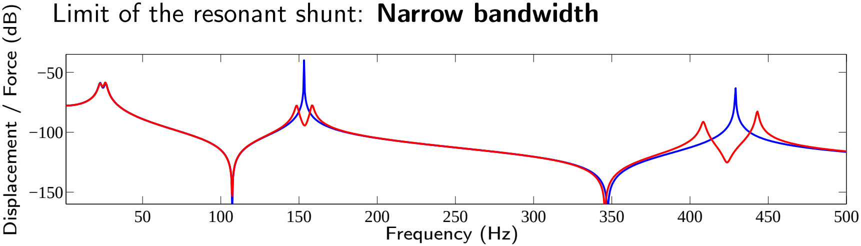
→ Multi-resonant network

Electrical analogue of the mechanical structure

→ **Multimodal damping** with a passive electrical network (✍ Porfiri, 2004)

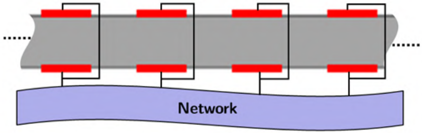
Passive technique for multimodal damping ?

Limit of the resonant shunt: **Narrow bandwidth**



Interconnected array

→ Multi-resonant network

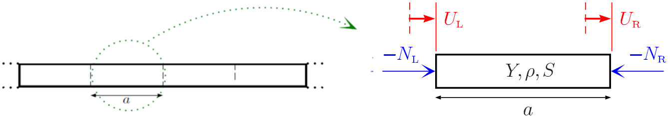


Electrical analogue of the mechanical structure

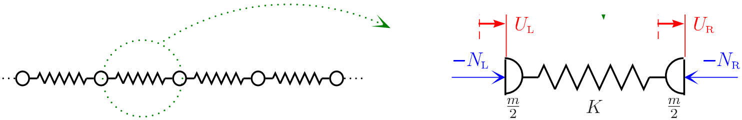
→ **Multimodal damping** with a passive electrical network (✍ Porfiri, 2004)

The analogous network is equivalent to the discrete model

Homogeneous rod for longitudinal wave propagation



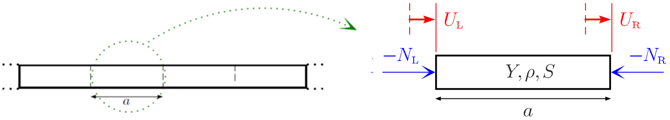
Corresponding **discrete structure** modeled by a lattice of point masses



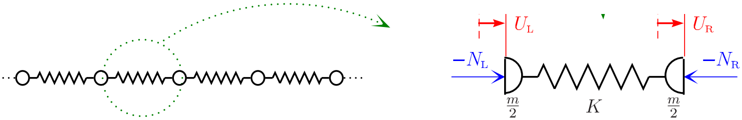
Analogous network involving capacitors and inductors (Direct analogy)

The analogous network is equivalent to the discrete model

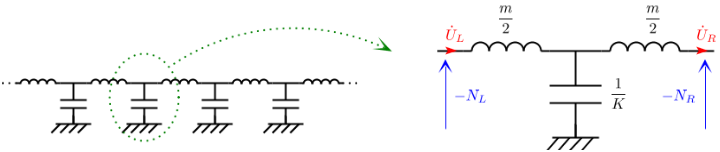
Homogeneous rod for longitudinal wave propagation



Corresponding **discrete structure** modeled by a lattice of point masses

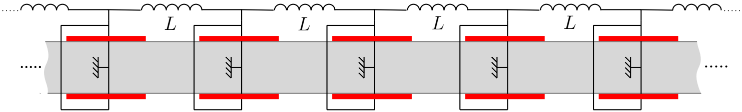


Analogous network involving capacitors and inductors (Direct analogy)



Interconnected patches enable multimodal damping

Array of piezoelectric patches → No external capacitors

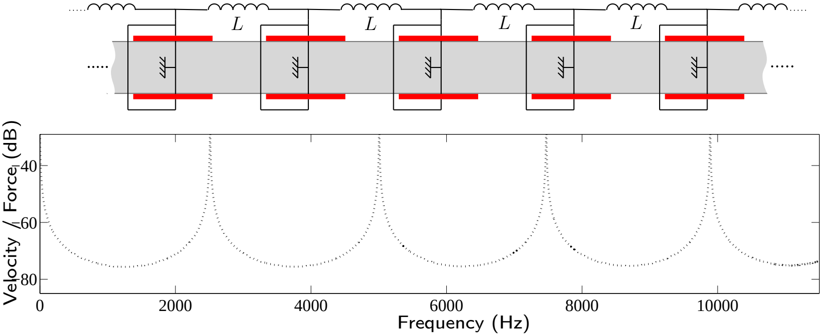


Same dispersion relations + Analogous boundary conditions

→ **"Multimodal tuned mass damper"**

Interconnected patches enable multimodal damping

Array of piezoelectric patches → No external capacitors



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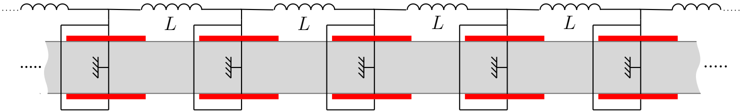

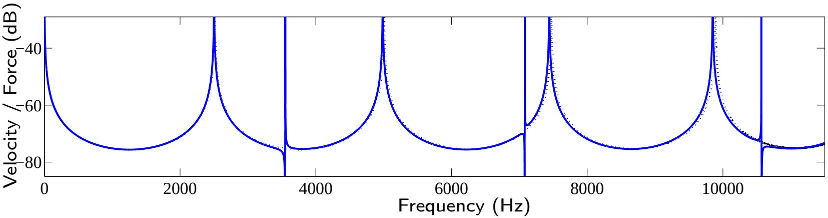

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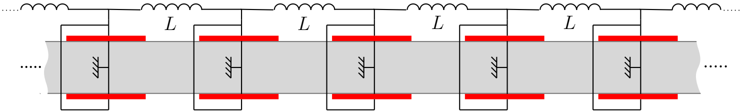

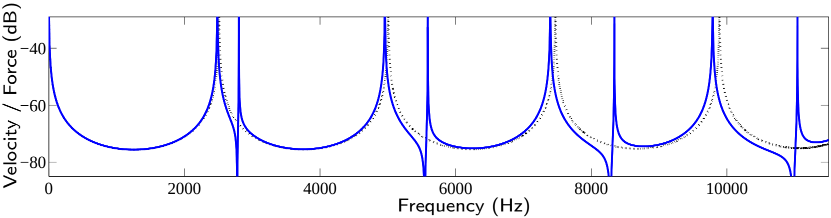

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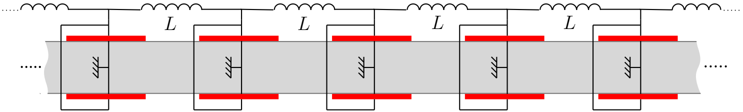

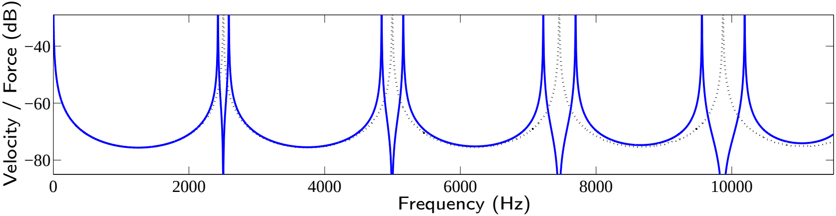

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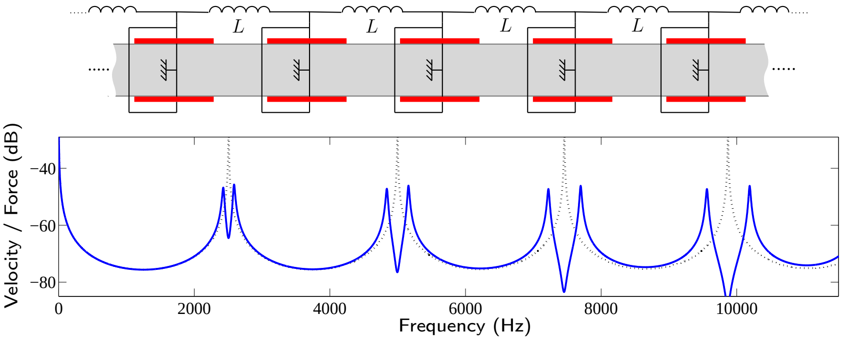

Same dispersion relations + Analogous boundary conditions

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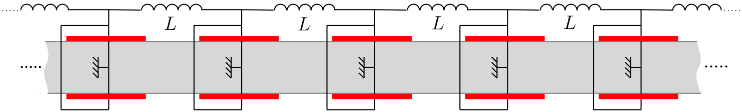
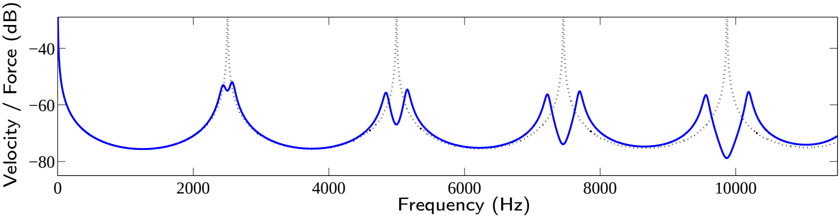
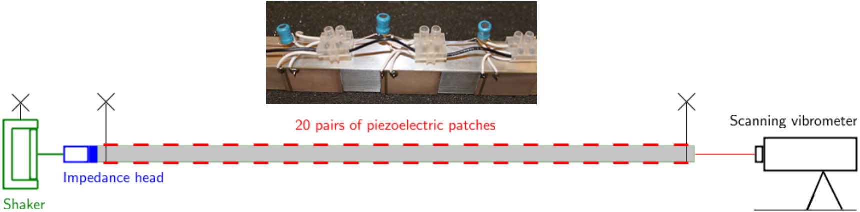




Same dispersion relations + Analogous boundary conditions

→ **"Multimodal tuned mass damper"**

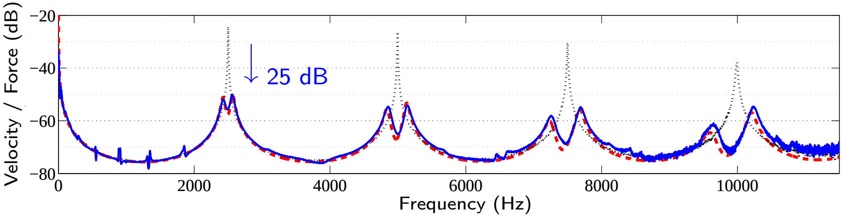
Models have been validated through experiments



First experimental validation of the control strategy

→ *B. Lossouarn, M. Aucejo, J.-F. Deü, Smart Materials & Structures, 2015*

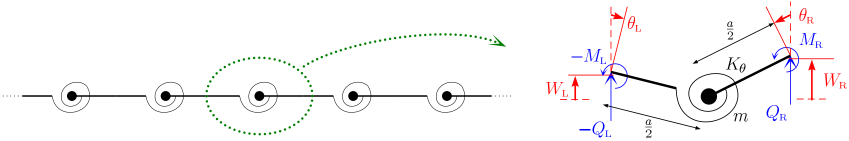
(…) Exp. without network, (—) Exp. with analogous network, (– –) Transfer matrix model



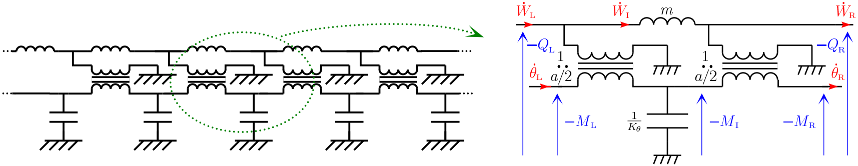
Discrete model for a beam

Same method: **Discrete model + Direct electromechanical analogy**

Beam approximated by a lattice involving the bending stiffness K_θ

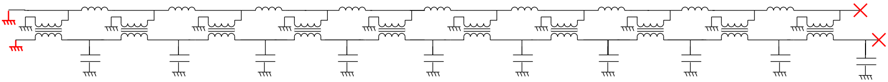


Analogous network with capacitors, inductors and transformers



Implementation of the analogous network

Analogous boundary conditions: Free = Short circuit, Clamped = Open circuit

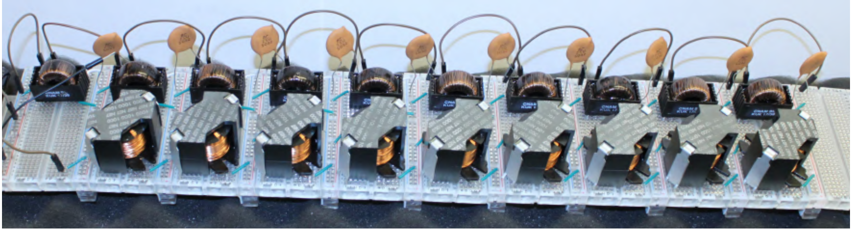


Design of **inductors** and **transformers** + Capacitors from standard series

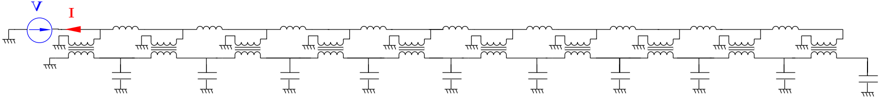
$\Rightarrow$ 9 $\times$

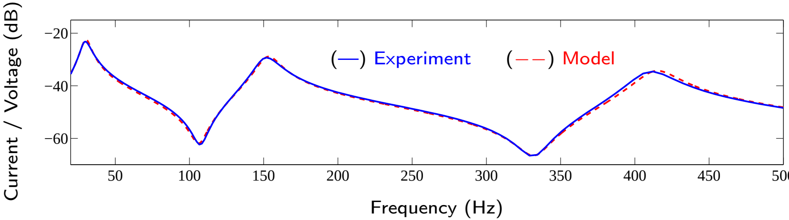
+ 10 $\times$

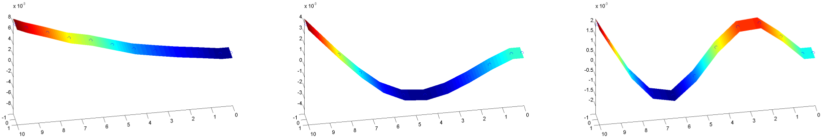
+ 10 $\times$



Admittance measurement at the "free end" of the electrical network



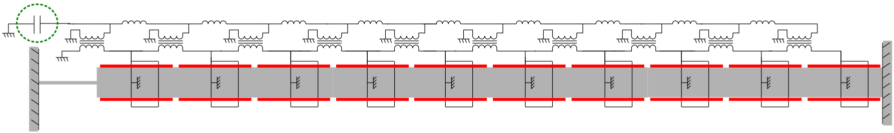


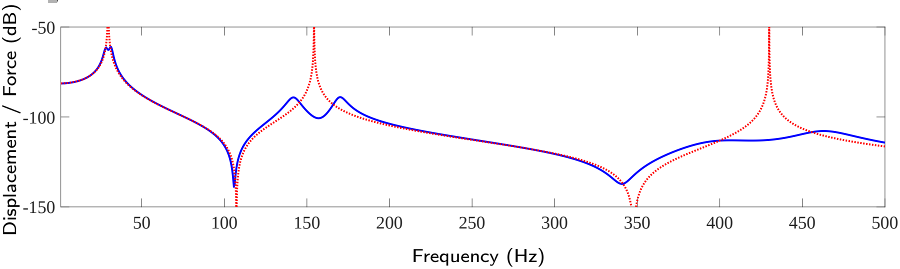


→ **Experimental modal analysis** of an electrical analogue

Broadband damping in the linear regime

Analogous end capacitor : $v = \frac{1}{C_{\mathrm{L}}} q$ where $C_{\mathrm{L}} = \frac{K_\theta}{K_{\mathrm{L}}} \frac{\hat{a}^2}{a^2} C_{\mathrm{p}}$

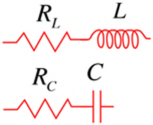




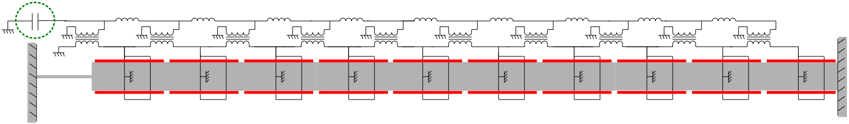
Optimal resistance for broadband damping ?

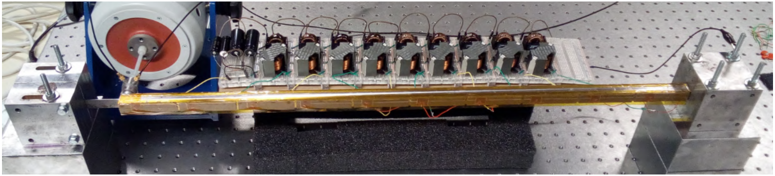
Lowest mode $\Rightarrow$ $Z_L = \mathrm{j}\omega L + R_L$

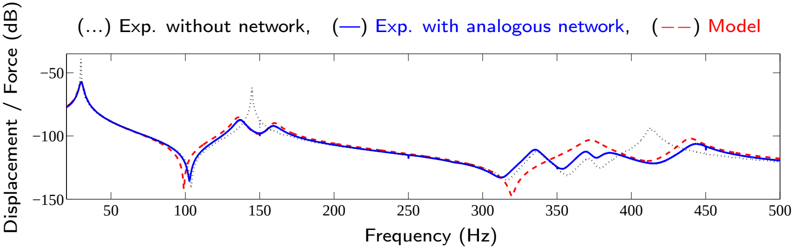
Highest mode $\Rightarrow$ $Z_C = \frac{1}{\mathrm{j}\omega C} + R_C$



Electromechanical coupling through piezoelectric patches



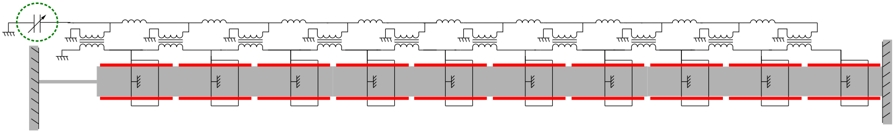




→ **Multimodal damping** observed on the mechanical response

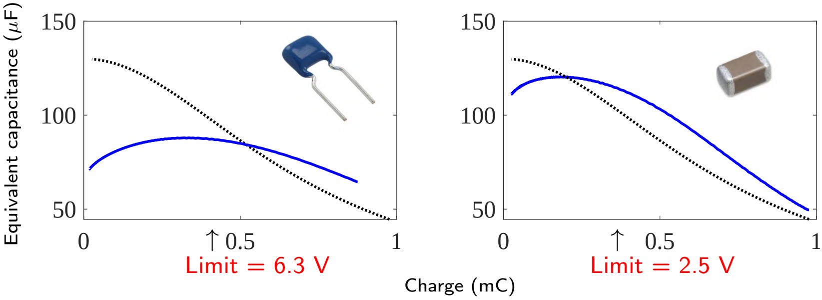
Toward a multimodal and nonlinear analogue

Variable electrical resonance due to variable capacitance



Nonlinear capacitor : $v = \frac{1}{C_{\mathrm{L}}} q + \frac{1}{C_{\mathrm{NL}}} q^3 \quad \Rightarrow \quad C(Q) \approx \frac{1}{\frac{1}{C_{\mathrm{L}}} + \frac{3Q^2}{4C_{\mathrm{NL}}}}$

$\rightarrow$ Solution = **Multilayer Ceramic Capacitor**



Kirchhoff-Love theory

$$-D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = \rho h \frac{\partial^2 w}{\partial t^2} \quad \Leftrightarrow \quad \begin{cases} \dfrac{\partial Q_x}{\partial x} + \dfrac{\partial Q_y}{\partial y} = -\rho h a \omega^2 W \\ Q_i = -\dfrac{\partial M_i}{\partial i}, \quad i = x, y \\ M = aD\left(\dfrac{\partial \theta_x}{\partial x} + \dfrac{\partial \theta_y}{\partial y}\right) \\ \theta_i = \dfrac{\partial W}{\partial i}, \quad i = x, y \end{cases}$$

→ New system of equations with **internal state variables Q, M and θ**

$\Downarrow$

Finite difference method based on a square plate unit cell

$$Q_R - Q_L + Q_T - Q_B = -m\omega^2 W_I$$
$$M_I = D\left(\theta_R - \theta_L + \theta_T - \theta_B\right)$$

+ Discrete derivatives

Extension to multimodal damping of plates

Kirchhoff-Love theory

$$-D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = \rho h \frac{\partial^2 w}{\partial t^2} \quad \Leftrightarrow \quad \begin{cases} \dfrac{\partial Q_x}{\partial x} + \dfrac{\partial Q_y}{\partial y} = -\rho h a \omega^2 W \\ Q_i = -\dfrac{\partial M_i}{\partial i}, \quad i = x, y \\ M = aD\left(\dfrac{\partial \theta_x}{\partial x} + \dfrac{\partial \theta_y}{\partial y}\right) \\ \theta_i = \dfrac{\partial W}{\partial i}, \quad i = x, y \end{cases}$$

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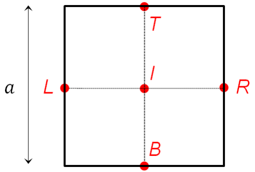


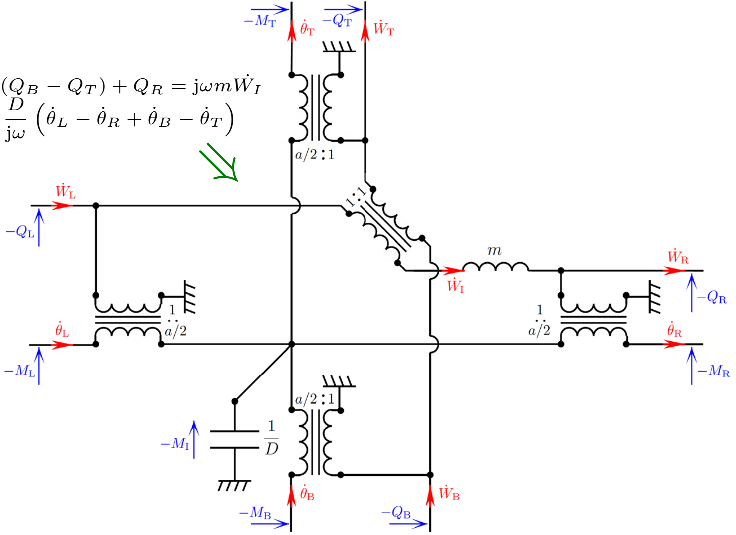
$$Q_R - Q_L + Q_T - Q_B = -m\omega^2 W_I$$
$$M_I = D\left(\theta_R - \theta_L + \theta_T - \theta_B\right)$$

+ Discrete derivatives

Obtain the analogous electrical topology

Direct electromechanical analogy: Force = Voltage and Velocity = Current

$$-Q_L - (Q_B - Q_T) + Q_R = \mathrm{j}\omega m \dot{W}_I$$
$$-M_I = \frac{D}{\mathrm{j}\omega}\left(\dot{\theta}_L - \dot{\theta}_R + \dot{\theta}_B - \dot{\theta}_T\right)$$



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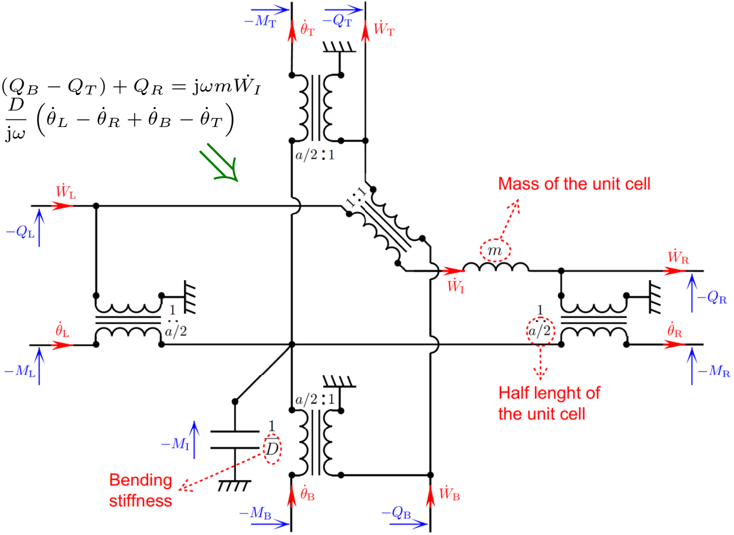
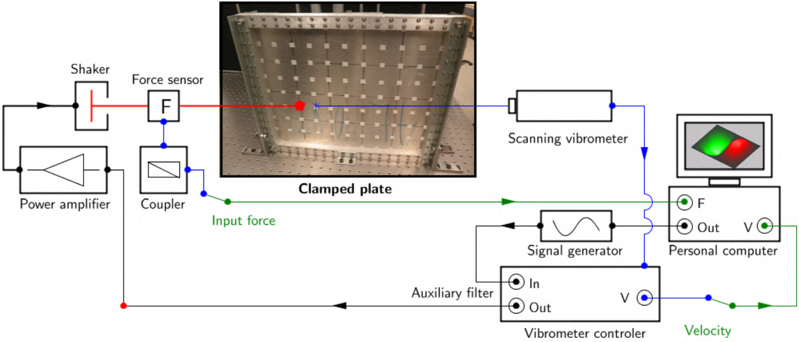
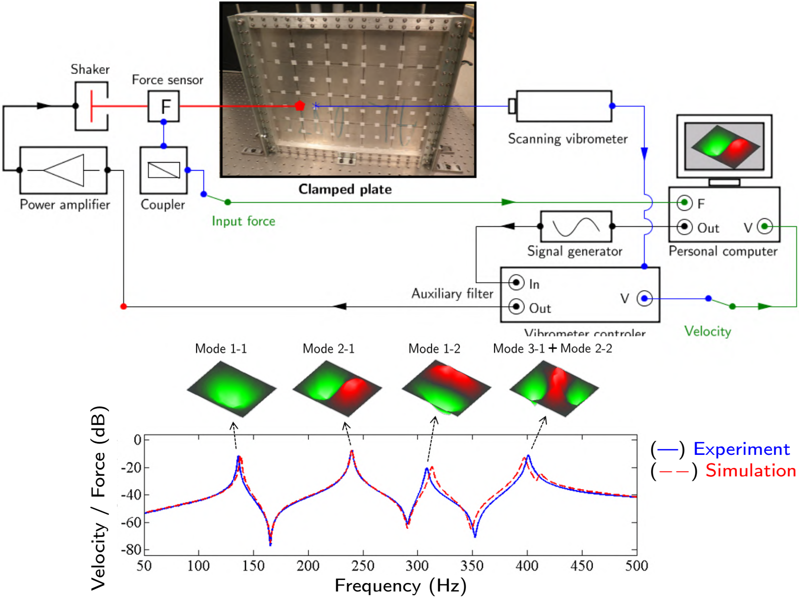


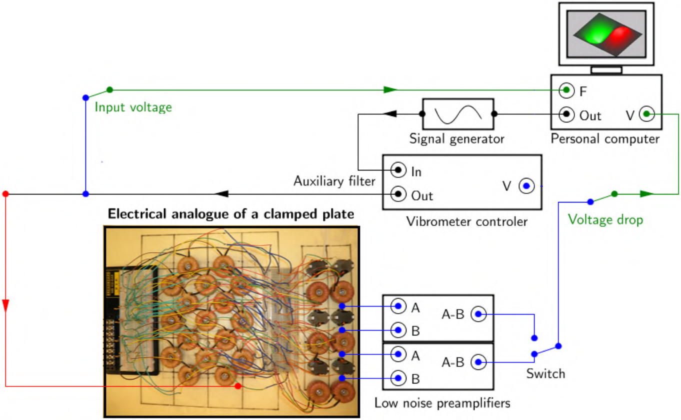
The network approximates the dynamics of a clamped plate



The network approximates the dynamics of a clamped plate



The network approximates the dynamics of a clamped plate



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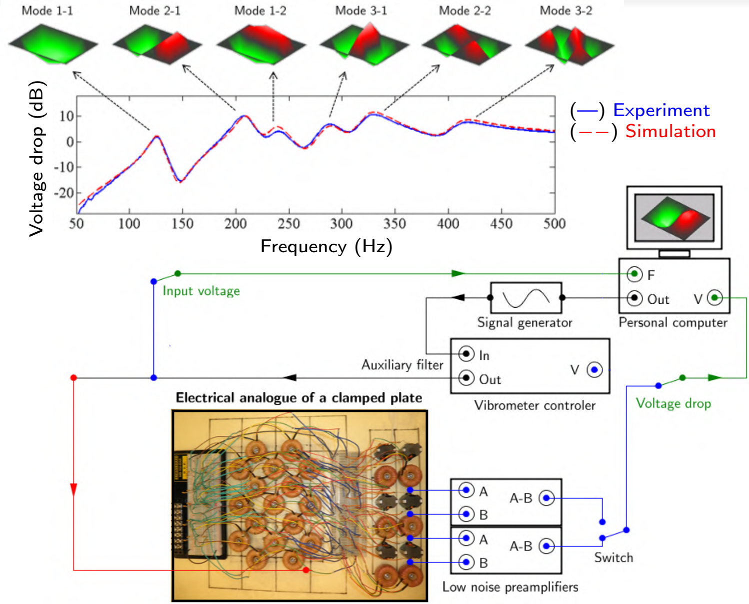
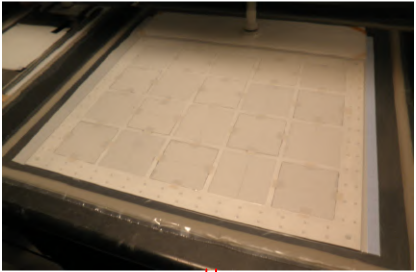


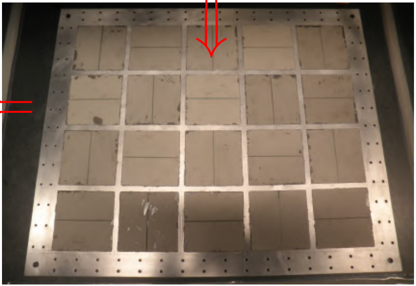
Vacuum bonding process

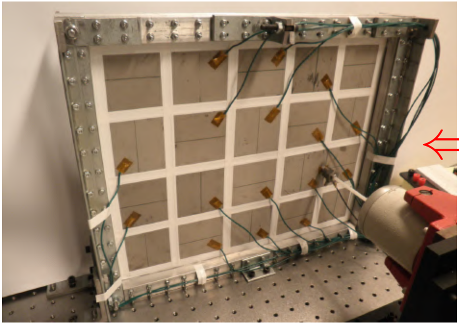
Aluminum plate
400×320 mm^2, 1.9 mm thick

20 PZT-5H square sheets
72.4×72.4 mm^2, 0.27 mm thick

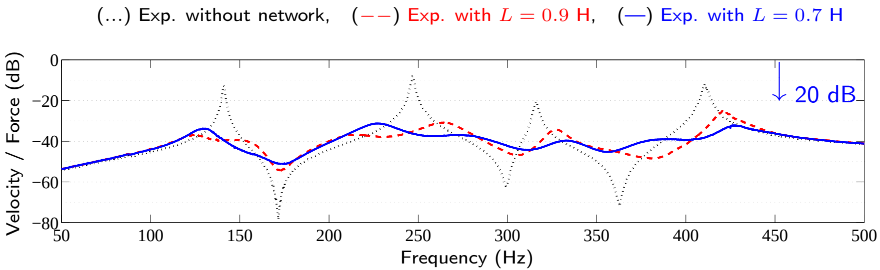
3M DP460 two-part epoxy







Analogous coupling generates broadband vibration reduction

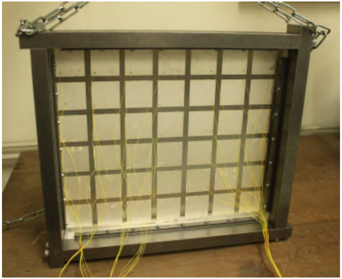


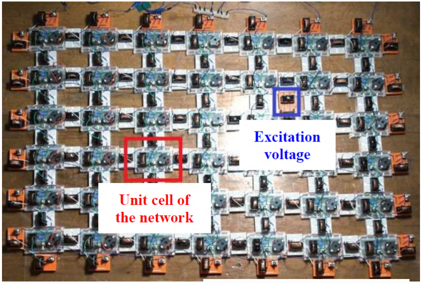
→ Experimental validation of the electromechanical model

→ **Multimodal tuned mass damping**

Broadband control of a continuous plate with a discrete network

Simply supported plate coupled to its analogous electrical network





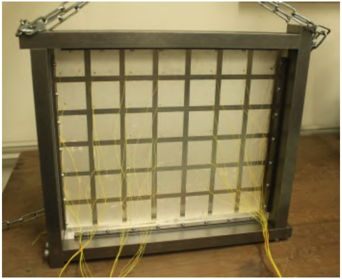
Current work

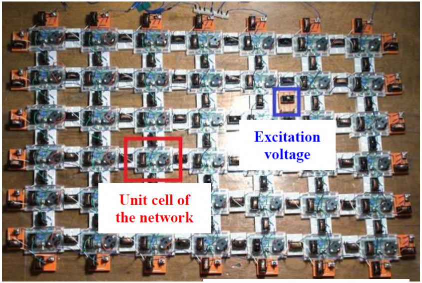
→ **Non-periodic** and complex structures

→ Comparison with **viscoelastic treatments**

Last plate setup at Cnam (see Robin Darleux...)

Simply supported plate coupled to its analogous electrical network





Current work

→ **Non-periodic** and complex structures

→ Comparison with **viscoelastic treatments**

Outline

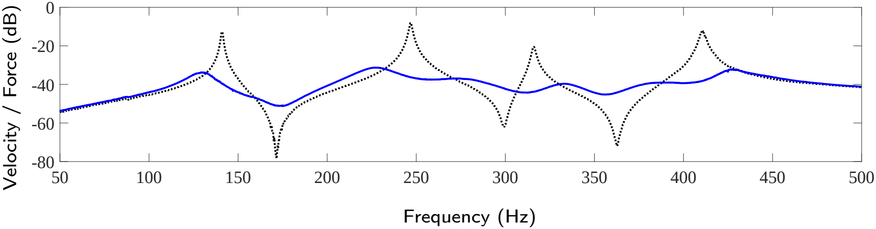
Passive damping with piezoelectric networks

Numerical models for complex electromechanical structures

Experimental validations

→ Multimodal damping with resonant electrical networks

→ **Passive, broadband and robust control strategy**



→ **Strong potential** for industrial applications

Thank you for your attention !

boris.lossouarn@lecnam.net